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ON THE CANONICAL HOLOMORPHIC MAP FROM THE MODULI SPACE OF STABLE CURVES TO THE IGUSA MONOIDAL TRANSFORM*)

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Introduction.

Let $\mathscr{M}_g$ be the coarse moduli space of complete non-singular curves of genus g and $\mathfrak{S}_g^*$ the coarse moduli space of principally polarized abelian varieties of dimension g. There is a canonical map:

$$i: \mathscr{M}_g \to \mathfrak{S}_g^*$$

defined by sending the isomorphism class of a curve C to the isomorphism class of the Jacobian variety of C. The famous theorem of Torelli asserts that this map i is injective (e.g. [28]). Moreover the map i is holomorphic (and even algebraic). It can be seen by rewriting the map i. That is, $\mathfrak{S}_g^*$ is defined analytically as the quotient space of the Siegel upper-half plane $\mathfrak{S}_g$ of degree g by the integral symplectic group $Sp(g, \boldsymbol{Z})$. It can be considered as the moduli space by letting Ω mod. $Sp(g, \boldsymbol{Z})$ correspond to the isomorphism class of $\boldsymbol{C}^g/(1_g, \Omega)\boldsymbol{Z}^{2g}$. Then the map i can be defined as the map which sends the isomorphism class of C to the residue class of the period matrix of C, and by this definition i is known to be holomorphic (cf. (4.1)).

However the spaces $\mathscr{M}_g$ and $\mathfrak{S}_g^*$ are not compact if $g > 0$, which gives rise to the problem of their compactification. Several kinds of compactifications with geometrical meaning are known. In case of $\mathscr{M}_g$ the moduli space $\mathscr{S}_g$ of stable curves of genus g due to Deligne and Mumford gives a good compactification ([4]). In case of $\mathfrak{S}_g^*$ the Satake compactification $\bar{\mathfrak{S}}_g^*$ is a natural one ([19], [20]). As a set $\bar{\mathfrak{S}}_g^*$ is a union of $\mathfrak{S}_{g'}^*$, $0 \leq g' \leq g$. However this compactification has too small boundary (of codimension g), so $\bar{\mathfrak{S}}_g^*$ is very singular at the boundary though normal. Igusa studied the desingularization problem

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