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STABLE VECTOR BUNDLES ON ALGEBRAIC SURFACES II

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This paper is a continuation of "Stable vector bundles on algebraic surfaces" [10]. For simplicity we deal with non-singular projective varieties over the field of complex numbers. Let W be a variety whose fundamental group is solvable, let H be an ample line bundle on W, and let $f: V \to W$ be an unramified covering. Then we show in section 1 that if E is an f^*H-stable vector bundle on V then f_*E is a direct sum of H-stable vector bundles. In particular f_*L is a direct sum of simple vector bundles if L is a line bundle on V. This result is a corollary of the following: Let A be a finite solvable group of automorphisms of a variety V. Suppose A acts freely on V. Let W be the quotient of V by A and let f be the natural morphism $V \to W$. Then the direct image of an f^*H-stable vector bundle on V by f is a direct sum of H-stable vector bundles, and the inverse image of an H-stable vector bundle on W by f is a direct sum of f^*H-stable vector bundles. In section 2 we prove the independence of H in the definition of the H-stability. Namely, let S be a relatively minimal surface, and let E be a vector bundle of rank two on S with $c_1^2(E) \geqq 4c_2(E)$. Then E is H-stable if and only if E is H'-stable, where H and H' are ample line bundles on S. We have proven this in our previous paper [10] in case $c_1^2(E) > 4c_2(E)$ without the assumption of relative minimality of S, and we obtained several results about H-stable vector bundles E with $c_1^2(E) = 4c_2(E)$ [10]. For instance, an H-stable vector bundle with $c_1^2 = 4c_2$ of rank two on an abelian surface is the direct image of a line bundle under an isogeny of a special type. And an H-stable vector bundle with $c_1^2 = 4c_2$ of rank two on a geometrically ruled surface is the vector bundle induced from a stable vector bundle on the base curve tensored with a line bundle on the surface. In connection with these results, we show in section 4 that on an elliptic bundle the vector bundle induced from a stable vector bundle of rank

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