0. Introduction. The purpose of this paper is to consider the following question: if \(R\) is a regular Noetherian ring and \(S \supseteq R\) is a module-finite \(R\)-algebra, is \(R\) a direct summand of \(S\) as \(R\)-modules? An affirmative answer is given if \(R\) contains a field, and it is shown that if the completions of the local rings of \(S\) possess maximal Cohen-Macaulay modules in the sense of §1 of [6] then the conclusion is valid in this case too. Hence, if Conjecture \(E\) of [6] is true then the question raised here has an affirmative answer without further restriction on the regular Noetherian ring \(R\), and it will be shown here that only a greatly weakened version of Conjecture \(E\) is needed.

It follows from our results on direct summands that if \(R\) contains a field, its local rings are regular, and \(S\) is an extension algebra integral over \(R\), then every ideal of \(R\) is contracted, i.e. if \(I \subset R\) then \(IS \cap R = I\). In fact, we prove the direct summand result by proving first that certain key ideals \(I\) of a regular local ring have this property. In the final section of the paper we consider briefly some propositions about the class of domains such that every ideal is contracted from every integral extension.

Throughout this paper, all rings are commutative, with identity, all modules are unitary, and ring homomorphisms are assumed to preserve the identity.

1. Regular rings. Our first reductions in the problem are consequences of the following:

**Lemma 1.** Let \(R \subseteq S\) be rings and assume that \(S\) is finitely presented as an \(R\)-module. Then \(R\) is a direct summand of \(S\) if and only if for