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IDEALS WITH SLIDING DEPTH

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Introduction

We study here a class of ideals of a Cohen-Macaulay ring $\{R, \mathfrak{m}\}$ somewhat intermediate between complete intersections and general Cohen-Macaulay ideals. Its definition, while a bit technical, rapidly leads to the development of its elementary properties. Let $I = (x_1, \cdots, x_n) = (\boldsymbol{x})$ be an ideal of R and denote by $H_*(\boldsymbol{x})$ the homology of the ordinary Koszul complex $K_*(\boldsymbol{x})$ built on the sequence $\boldsymbol{x}$. It often occurs that the depth of the module H_i, $i > 0$, increases with i (as usual, we set depth $(0) = \infty$). We shall say that I satisfies *sliding depth* if

$$\text{(SD)} \qquad \operatorname{depth} H_i(\boldsymbol{x}) \geq \dim(R) - n + i, \quad i \geq 0.$$

This definition depends solely on the number of elements in the sequence $\boldsymbol{x}$. This property localizes (cf. [9]) and is an invariant of even linkage (cf. [10]).

An extreme case of this property is given by a complete intersection. A more general instance of it is that where all the modules H_i are Cohen-Macaulay, a situation that was dubbed *strongly* Cohen-Macaulay ideals (cf. [11]).

These ideals have appeared earlier in two settings:

(i) The investigation of arithmetical properties of the Rees algebra of I

$$S = \mathscr{R}(I) = \oplus I^s,$$

and of the associated graded ring

$$G = \operatorname{gr}_I(R) = \oplus I^s/I^{s+1}.$$

It was shown in [7], [8] and [16] that for ideals satisfying (SD) and such that for each prime P containing I, height(P)=ht$(I) \geq v(I_p)$ = minimum number of generators of the localization I_p, both S and G are Cohen-Macaulay. In addition, if R is a Gorenstein ring, G will be Goren-

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