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Nagoya Math. J.
Vol. 99 (1985), 147–157

MATHIEU GROUP M_{24} AND MODULAR FORMS

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§ 0. Introduction

In [6], Mason reported some connections between sporadic simple group M_{24} and certain cusp forms which appear in the 'denominator' of Thompson series assigned to Fisher-Griess's group F_1. In this paper, we discuss the 'numerator' of these Thompson series.

We state our result precisely. Since M_{24} is a subgroup of the symmetric group S_{24} of degree 24, we can write for each $m \in M_{24}$,

$$m = (n_1)(n_2) \cdots (n_s), \qquad n_1 \geqq \cdots \geqq n_s \geqq 1,$$

to mean that m is a product of cycle of length n_i, $1 \leqq i \leqq s$. To each $m = (n_1) \cdots (n_s)$, we associate modular forms $\eta_m(z)$ and $\vartheta_m(z)$ as follows; let

$$\eta_m(z) = \eta(n_1 z) \cdots \eta(n_s z),$$

where $\eta(z)$ is the Dedekind η-function

$$\eta(z) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n),$$

where $q = \exp(2\pi\sqrt{-1}z)$ and $z \in \boldsymbol{H} = \{z \in \boldsymbol{C} \mid \operatorname{Im} z > 0\}$. Then, in [6], Mason showed that $\eta_m(z)$ is a cusp form of weight $s/2$ on $\Gamma_0(n_1 n_s)$ with some character ε_m and is also a common eigenfunction of all Hecke operators, (see also [3]).

On the other hand, it is well-known that M_{24} acts on the Leech lattice L as isometries. To each $m \in M_{24}$, put

$$L^m = \{x \in L \mid m \cdot x = x\}.$$

Then L^m is an even integral, positive definite quadratic lattice of rank s. Let $\vartheta_m(z)$ denote the theta function of L^m:

Received June 15, 1984.