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THE FUNCTIONAL EQUATION OF ZETA DISTRIBUTIONS ASSOCIATED WITH PREHOMOGENEOUS VECTOR SPACES $(\tilde{G}, \tilde{\rho}, M(n, C))$

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Introduction

Let (G, ρ, V) be a triple of a linear algebraic group G and a rational representation ρ on a finite dimensional vector space V, all defined over the complex number field $\boldsymbol{C}$.

We call the triple (G, ρ, V) a prehomogeneous vector space if G has a Zariski-open orbit. Assume that the triple (G, ρ, V) is a prehomogeneous vector space. Then there exists a proper algebraic subset S of V such that $V-S$ is a single G-orbit. The algebraic set S is called the singular set of (G, ρ, V). For a rational character of G, a non-zero rational function P on V is called a relative invariant of (G, ρ, V) corresponding to χ if

$$P(\rho(g)x) = \chi(g)P(x) \qquad (g \in G,\ x \in V).$$

Let $P_1, \cdots, P_n$ be irreducible polynomials defining the components of S with codimension 1. It is known that $P_1, \cdots, P_n$ are relative invariants of (G, ρ, V) (cf. [1]). The set $\{P_1, \cdots, P_n\}$ is called a complete set of irreducible relative invariants of (G, ρ, V).

The purpose of this paper is to give an explicit expression for the Fourier transform of relative invariants on a certain class of prehomogeneous vector spaces.

NOTATION. We denote by $\boldsymbol{Z}$, $\boldsymbol{R}$ and $\boldsymbol{C}$ the ring of integers, the rational number field and the complex number field, respectively. For $z \in \boldsymbol{C}$, we set $e(z) = \exp(2\pi\sqrt{-1}z)$. We denote by $M(n, \boldsymbol{C})$ (resp. $M(n, \boldsymbol{R})$) the complex (resp. real) vector space consisting of all n by n matrices with entries in $\boldsymbol{C}$ (resp. $\boldsymbol{R}$). For any matrix x, ${}^t x$ denotes the transposed matrix. For

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