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THE SEMI-BALAYABILITY OF REAL CONVOLUTION KERNELS

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Dedicated to Professor Yukio Kusunoki on his 60th birthday

§1.

Let X be a locally compact, σ-compact and non-compact abelian group. Throughout this paper, we shall denote by ξ a fixed Haar measure on X and by δ the Alexandroff point of X.

A real convolution kernel (i.e., a real Radon measure) N on X is said to be semi-balayable if N satisfies the semi-balayage principle on all open sets (see Definition 6). We know that every convolution kernel N of logarithmic type is semi-balayable (see [8]). Here N is said to be of logarithmic type if, with a vaguely continuous, markovian, semi-transient and recurrent convolution semi-group $(\alpha_t)_{t\geqq 0}$ of non-negative Radon measures on X,

$$N * \mu = \int_0^\infty \alpha_t * \mu dt \left(= \lim_{t\to\infty} \int_0^t \alpha_s * \mu ds\ ^{1)}\right)$$

for all real Radon measure μ on X with compact support and $\int d\mu = 0$.

In this paper, we shall show that the semi-balayability is an essential property to characterize convolution kernels of logarithmic type. More precisely, we shall establish the following theorems.

THEOREM 1. *Let N be a real convolution kernel on X. If $X \approx R \times F$ or $X \approx Z \times F$, we suppose an additional condition: $N = o(|x|)$ at the infinity*[2]. *Then N is of logarithmic type if and only if N is semi-balayable, non-periodic and satisfies $\inf_{x\in X} N * f(x) \leqq 0$ for any finite continuous function f on X with compact support and $\int f d\xi = 0$.*

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1) For a net $(\mu_\alpha)_{\alpha\in A}$ of real Radon measures and a real Radon measure μ, we write $\mu = \lim_{\alpha\in A} \mu_\alpha$ if $(\mu_\alpha)_{\alpha\in A}$ converges vaguely to μ along A.

2) If $X = R \times F$ or $X = Z \times F$, $N = o(|x|)$ at the infinity means that for any $f \in C_K^+(X)$, $N * f((x, y)) = o(|x|)$ as $|x| \to \infty$, where $(x, y) \in R \times F$ or $\in Z \times F$. In the case of $X \approx R \times F$ or $X \approx Z \times F$, the definition $N = o(|x|)$ at the infinity follows naturally from the above definition.