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SOME RESULTS ON HARMONIC ANALYSIS ON COMPACT QUOTIENTS OF HEISENBERG GROUPS

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Heisenberg group $H_{2g+1}(\boldsymbol{R})$ of dimension $2g+1$ is a real nilpotent group defined on $\boldsymbol{R}\times\boldsymbol{R}^g\times\boldsymbol{R}^g$ by the law of composition

$$(x_0, \hat{x}, x)\circ(y_0, \hat{y}, y) = (x_0 + y_0 + \hat{x}^t y, \hat{x}+\hat{y}, x+y),$$

which is isomorphic to the unipotent matrix group

$$\left\{\begin{pmatrix} 1 & \hat{c}_1, \cdots, \hat{c}_g, & c_0 \\ & 1 & c_1 \\ & \ddots & \vdots \\ & 1 & c_g \\ & & 1 \end{pmatrix}\right\} \quad (c_0, \hat{c}_i, c_i \in \boldsymbol{R}, 1\leqslant i, j\leqslant g).$$

$H_{2g+1}(\boldsymbol{Z})$ means the discrete subgroup of integral elements, and $L^2(H_{2g+1}(\boldsymbol{Z})\backslash H_{2g+1}(\boldsymbol{R}))$ is the L^2-space of the quotient space

$$H_{2g+1}(\boldsymbol{Z})\backslash H_{2g+1}(\boldsymbol{R})$$

with respect to the invariant measure

$$dx_0 d\hat{x} dx = dx_0 d\hat{x}_1 \cdots d\hat{x}_g dx_1 \cdots dx_g .$$

The right action of $H_{2g+1}(\boldsymbol{R})$ induces a unitary representation ρ on $L^2(H_{2g+1}(\boldsymbol{Z})\backslash H_{2g+1}(\boldsymbol{R}))$:

$$\rho(y_0, \hat{y}, y)\phi(x_0, \hat{x}, x) = \phi((x_0, \hat{x}, x)\circ(y_0, \hat{y}, y)).$$

For each non-zero real number λ $H_{2g+1}(\boldsymbol{R})$ also acts on the usual L^2-space $L^2(\boldsymbol{R}^g)$ as follows

$$\chi_\lambda(y_0, \hat{y}, y)f(\xi) = \exp(2\pi\lambda\sqrt{-1}(y_0 + \hat{y}^t\xi))f(\xi+y),$$
$$(f(\xi)\in L^2(\boldsymbol{R}^g), \qquad (y_0, \hat{y}, y)\in H_{2g+1}(\boldsymbol{R})),$$

Since Lebesgue measure is invariant with respect to translations, χ_λ is a

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