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RINGS OF MODULAR FORMS ON EICHLER'S PROBLEM

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In his paper [4] or lecture note [3], Eichler asked the problem when the ring of modular forms is Cohen-Macaulay. We shall try to investigate it for the Hilbert or Siegel modular case.

When the dimension n is one, any ring of modular forms for an arithmetic group is Cohen-Macaulay, indeed a normal (graded) ring of Krull dimension two is always Cohen-Macaulay. So we consider the case $n > 1$. Unfortunately rings of modular forms do not always have this nice property. In the case of (symmetric or not) Hilbert modular forms it is essentially Freitag's result (see 7.1 Satz [6] and Proposition A in § 1.1). Let $A(\Gamma) = \oplus_k A(\Gamma)_k$ be the ring of Hilbert modular forms for a group Γ. Then the same question for $A(\Gamma)^{(2)} = \oplus_{k\equiv 2(0)} A(\Gamma)_k$ with $n = 2$ was raised by Thomas and Vasquez [20], in which it is shown by using the criterion due to Stanley [18], [19] that $A(\Gamma)^{(2)}$ is also Gorenstein if it is Cohen-Macaulay under some condition on Γ. Also Eichler derived some consequence of the 'hypothesis' of $A(\Gamma)^{(2)}$ being Cohen-Macaulay with $n = 2$ in [3].

In this paper we shall show this affirmatively, and moreover get when $A(\Gamma)^{(r)}$ is Cohen-Macaulay for general n and $r \geq 2$, as well as the case of symmetric Hilbert modular forms (Theorem 1). Furthermore if $n = 2$ and if Γ acts freely on H^2, the necessary and sufficient condition for $A(\Gamma)$ to be Cohen-Macaulay is given as

$$(1) \qquad \dim A(\Gamma)_1 = \tfrac{1}{2}(-\tfrac{1}{2}\zeta_K(-1)\cdot a + \chi + h)$$

where K is a corresponding real quadratic field, ζ_K is its zeta function, $a = [SL_2(O_K); \Gamma]$, O_K being the ring of integers of K, h is the number of the cusps and χ is the arithmetic genus of the non-singular model of the Hilbert modular surface.

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