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COUNTABLE J_a^S-ADMISSIBLE ORDINALS

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§ 0. Introduction

In [3], Platek constructs a hierarchy of jumps J_a^S indexed by elements a of a set $\mathcal{O}^S$ of ordinal notations. He asserts that a real $X \subseteq \omega$ is recursive in the superjump S if and only if it is recursive in some J_a^S. Unfortunately, his assertion is not correct as is shown in [1]. In [1], it also has been shown that an ordinal $> \omega$ is J_a^S-admissible if it is $|a|_S$-recursively inaccessible, where $|a|_S$ is the ordinal denoted by a.

Let A be an arbitrary set. We say that an oridinal α is A-admissible if the structure $\langle L_\alpha[A], \in, A \cap L_\alpha[A]\rangle$, which we will denote by $L_\alpha[A]$ for simplicity, is admissible, a model of the Kripke-Platek set theory KP, where $L_\alpha[A]$ is the sets constructible relative to A in fewer than α steps. We use ω_1^A or $\omega_1(A)$ to denote the first A-admissible ordinal $> \omega$, and use $\omega_1(A_1, \cdots, A_n)$ for $\omega_1(\langle A_1, \cdots, A_n\rangle)$.

The purpose of this paper is to prove the following theorem.

THEOREM 1. *Suppose $a \in \mathcal{O}^S$ and $\alpha > \omega$ is a countable $|a|_S$-recursively inaccessible ordinal. Then, there exists a real $X \subseteq \omega$ such that $\alpha = \omega_1(J_a^S, X)$.*

In the case $|a|_S = 0$, $J_a^S = {}^2E$, the Kleene object of type 2, and $\omega_1({}^2E, X) = \omega_1^X$ for all reals $X \subseteq \omega$. α is an admissible oridnal if and only if it is 0-recursively inaccessible. Therefore, Theorem 1 is an extension of the following theorem of Sacks.

THEOREM 2 (Sacks [4]). *If $\alpha > \omega$ is a countable admissible ordinal, then there exists a real X such that $\alpha = \omega_1^X$.*

Sacks also showed that the real X mentioned in Theorem 2 can be taken to have the minimality property:

$$\omega_1^Y < \alpha \text{ for every } Y \text{ of lower hyperdegree than } X.$$

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