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RELATIVE INVARIANTS AND b-FUNCTIONS OF PREHOMOGENEOUS VECTOR SPACES

$(G \times GL(d_1, \cdots, d_r), \tilde{\rho}_1, M(n, C))$

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Introduction

Let G be a connected linear algebraic group, ρ a rational representation of G on a finite-dimensional vector space V, all defined over C.

A polynomial $f(x)$ on V is called a relative invariant, if there exists a rational character $\chi : G \to C^{\times}$ satisfying

$$f(\rho(g)\cdot x) = \chi(g)f(x)\,, \qquad \text{for any } g \in G \text{ and } x \in V\,.$$

The triplet (G, ρ, V) is called a prehomogeneous vector space (abbrev. P.V.), if there exists a proper algebraic subset S of V such that $V - S$ is a single G-orbit. The algebraic set S is called the singular set of (G, ρ, V) and any point in $V - S$ is called a generic point of (G, ρ, V).

Let $GL(d_1, \cdots, d_r)$ be a parabolic subgroup of the general linear group $GL(n, C)$ defined by (1.1) in Section 1, $\rho : G \to GL(n, C)$ be an n-dimensional representation of G. In this paper, we shall be concerned with the triplet $(G \times GL(d_1, \cdots, d_r), \tilde{\rho}_1, M(n, C))$, where $\tilde{\rho}_1$ is defined by

$$\rho_1(g, a)x = \rho(g)xa^{-1} \quad ((g, a) \in G \times GL(d_1, \cdots, d_r),\ x \in M(n, C))\,.$$

Assume that $(G \times GL(d_1, \cdots, d_r), \tilde{\rho}_1, M(n, C))$ is a P.V. We shall introduce the b-function of $(G \times GL(d_1, \cdots, d_r), \tilde{\rho}_1, M(n, C))$, after M. Sato, in Section 3. Theorem 3.1 gives an explicit form of the b-function. In Section 4, we shall be concerned with triplets $\{(G\times B_n, \tilde{\rho}_1, M(n, C))\}$ where G is a semi-simple connected linear algebraic group, B_n is the upper triangular group and ρ is an irreducible representation on an n-dimensional vector space V. We shall determine all prehomogeneous vector space $\{(G \times B_n, \tilde{\rho}_1, M(n, C)\}$, and construct their relative invariants.

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