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ON A GENERALIZATION OF HAMBURGER'S THEOREM

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Introduction

The relationship between Poisson's summation formula and Hamburger's theorem [2] which is a characterization of Riemann's zetafunction by the functional equation was already mentioned in Ehrenpreis-Kawai [1]. There Poisson's summation formula was obtained by the functional equation of Riemann's zetafunction. This procedure is another proof of Hamburger's theorem. Being interpreted in this way, Hamburger's theorem admits various interesting generalizations, one of which is to derive, from the functional equations of the zetafunctions with Grössencharacters of the Gaussian field, Poisson's summation formula corresponding to its ring of integers [1]. The main purpose of the present paper is to give a generalization of Hamburger's theorem to some zetafunctions with Grössencharacters in algebraic number fields. More precisely, we first define the zetafunctions with Grössencharacters corresponding to a lattice in a vector space, and show that Poisson's summation formula yields the functional equations of them. Next, we derive Poisson's summation formula corresponding to the lattice from the functional equations.

§1. Notations and formulation of the theorem

We denote by $\boldsymbol{R}$ and $\boldsymbol{C}$ the field of real numbers and the field of complex numbers respectively. Let F be an algebraic number field of degree n with signature $[r_1, r_2]$. We can naturally embed F into $\boldsymbol{R}^{r_1} \times \boldsymbol{C}^{r_2}$. Put $V = \boldsymbol{R}^{r_1} \times \boldsymbol{C}^{r_2}$. Then V may be regarded as a commutative ring. For $x = (x^{(1)}, \cdots, x^{(r_1+r_2)}) \in V$, we put $trx = \sum_{p=1}^{r_1+r_2} tr_{\boldsymbol{R}} x^{(p)}$, $Nx = \prod_{p=1}^{r_1+r_2} |N_{\boldsymbol{R}} x^{(p)}|$, $e(x) = \exp(2\pi\sqrt{-1}\, trx)$. We define a lattice in V as a subgroup of V having a basis $\{\alpha_1, \cdots, \alpha_n\}$ independent over $\boldsymbol{R}$. Let L be a lattice in V. Then its dual can be defined as the set L^* consisting of all $x \in V$ such that $e(xy) = 1$ for all $y \in L$. We can show easily that L^*

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