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ON 3-DIMENSIONAL TERMINAL SINGULARITIES

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Introduction

Canonical and terminal singularities are introduced by M. Reid [5], [6]. He proved that 3-dimensional terminal singularities are cyclic quotient of smooth points or cDV points [6].

Let (X, p) be a 3-dimensional terminal singularity of index m with the associated Z_m-cover $(\tilde{X}, \tilde{p}) \to (X, p)$. If (X, p) is a cyclic quotient singularity (i.e. if $(\tilde{X}, \tilde{p})$ is smooth), then it is known as Terminal Lemma (Danilov [3], D. Morrison-G. Stevens [4]) that there exist an integer a prime to m and coordinates x, y, z of $(\tilde{X}, \tilde{p})$ which are Z_m-semi-invariants such that $\sigma(x) = \zeta x$, $\sigma(y) = \zeta^{-1} y$, $\sigma(x) = \zeta^a z$ for the standard generator σ of Z_m, where ζ is a primitive m-th root of 1. In this paper, we consider the case where $(\tilde{X}, \tilde{p})$ is a singular point and $m > 1$. The main results are Theorems 12, 23, 25 and Remarks 12.2, 23.1, 25.1. These, together with the Terminal Lemma above, almost classify 3-dimensional terminal singularities.

Since $(\tilde{X}, \tilde{p})$ is an isolated singularity (or smooth) and is a hypersurface defined by a Z_m-semi-invariant power series (say φ), all deformations of (X, p) are induced by deformations of φ as a Z_m-semi-invariant power series [2, §§ 9–10]. By Theorems 12, 23 and 25, one can see that there is a semi-invariant coordinate which has the same character as φ (e.g. z in Theorem 12, (1)), and hence every terminal singularity can be deformed to a cyclic quotient singularity (e.g. by $\varphi + \lambda z$ with parameter λ for the case Theorem 12, (1)). This is not necessarily the case with canonical singularities.

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As for the notation, we say that a monomial (say u^2) appears in a

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