ON 3-DIMENSIONAL TERMINAL SINGULARITIES

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Introduction

Canonical and terminal singularities are introduced by M. Reid [5], [6]. He proved that 3-dimensional terminal singularities are cyclic quotient of smooth points or cDV points [6].

Let \((X, p)\) be a 3-dimensional terminal singularity of index \(m\) with the associated \(\mathbb{Z}_m\)-cover \((\tilde{X}, \tilde{r}) \to (X, p)\). If \((X, p)\) is a cyclic quotient singularity (i.e. if \((\tilde{X}, \tilde{p})\) is smooth), then it is known as Terminal Lemma (Danilov [3], D. Morrison-G. Stevens [4]) that there exist an integer \(a\) prime to \(m\) and coordinates \(x, y, z\) of \((\tilde{X}, \tilde{p})\) which are \(\mathbb{Z}_m\)-semi-invariants such that \(\sigma(x) = \zeta x, \sigma(y) = \zeta^{-1} y, \sigma(z) = \zeta^az\) for the standard generator \(\sigma\) of \(\mathbb{Z}_m\), where \(\zeta\) is a primitive \(m\)-th root of 1. In this paper, we consider the case where \((\tilde{X}, \tilde{p})\) is a singular point and \(m > 1\). The main results are Theorems 12, 23, 25 and Remarks 12.2, 23.1, 25.1. These, together with the Terminal Lemma above, almost classify 3-dimensional terminal singularities.

Since \((\tilde{X}, \tilde{p})\) is an isolated singularity (or smooth) and is a hypersurface defined by a \(\mathbb{Z}_m\)-semi-invariant power series (say \(\varphi\)), all deformations of \((X, p)\) are induced by deformations of \(\varphi\) as a \(\mathbb{Z}_m\)-semi-invariant power series [2, §§9–10]. By Theorems 12, 23 and 25, one can see that there is a semi-invariant coordinate which has the same character as \(\varphi\) (e.g. \(z\) in Theorem 12, (1)), and hence every terminal singularity can be deformed to a cyclic quotient singularity (e.g. by \(\varphi + \lambda z\) with parameter \(\lambda\) for the case Theorem 12, (1)). This is not necessarily the case with canonical singularities.

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As for the notation, we say that a monomial (say \(u^r\)) appears in a