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ON THE NUMBER OF DIFFEOMORPHISM CLASSES IN A CERTAIN CLASS OF RIEMANNIAN MANIFOLDS

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§ 0. Introduction

The study of finiteness for Riemannian manifolds, which has been done originally by J. Cheeger [5] and A. Weinstein [13], is to investigate what bounds on the sizes of geometrical quantities imply finiteness of topological types, —e.g. homotopy types, homeomorphism or diffeomorphism classes— of manifolds admitting metrics which satisfy the bounds. For a Riemannian manifold M we denote by R_M and K_M respectively the curvature tensor and the sectional curvature, by $\mathrm{Vol}(M)$ the volume, and by $\mathrm{diam}(M)$ the diameter.

CHEEGER'S FINITENESS THEOREM I [5]. *For given n, Λ, $V > 0$ there exist only finitely many pairwise non-diffeomorphic (non-homeomorphic) closed $n(\neq 4)$-manifolds (4-manifolds) which admit metrics such that* $|K_M| \leq \Lambda^2$, $\mathrm{diam}(M) \leq 1$, $\mathrm{Vol}(M) \geq V$.

He proved directly finiteness up to homeomorphism for all dimension, and then for $n \neq 4$ used the results of Kirby and Siebenmann which show that finiteness up to homeomorphism implies finiteness up to diffeomorphism. For $n = 4$, he put a stronger bound on $\|\nabla R\|$, where ∇R denotes the covariant derivative of curvature tensor R. For given n, Λ, Λ_1, $V > 0$, we denote by $\mathfrak{M}^n(\Lambda, \Lambda_1, V)$ a class of closed n-dimensional Riemannian manifolds M which satisfy the following bounds;

$$|K_M| \leq \Lambda^2, \quad \|\nabla R_M\| \leq \Lambda_1, \quad \mathrm{diam}(M) \leq 1, \quad \mathrm{Vol}(M) \geq V,$$

and set $\mathfrak{M}(\Lambda, \Lambda_1, V) = \bigcup_n \mathfrak{M}^n(\Lambda, \Lambda_1, V)$.

CHEEGER'S FINITENESS THEOREM II [5]. *For given n, Λ, Λ_1, $V > 0$, the number $\sharp_{\mathrm{diff}}\, \mathfrak{M}^n(\Lambda, \Lambda_1, V)$ of diffeomorphism classes in $\mathfrak{M}^n(\Lambda, \Lambda_1, V)$ is finite.*

In the proof of the Cheeger finiteness theorem and our results as

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