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# ON THE RANK OF CM-TYPE

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In the present note, we prove that every simple CM-type is non-degenerate (i.e. the rank is maximal) if the dimension of corresponding abelian varieties is a prime. This follows directly from the argument of Tankeev [5], in which he has treated the 5-dimensional case.

Recently, S. G. Tankeev and K. A. Ribet have established similar results for more general types of abelian varieties (see [3], [4], [6]).

**1.** Let $K$ be a CM-field (i.e. a totally imaginary quadratic extension of a totally real number field). We regard $K$ as a subfield of $\boldsymbol{C}$. Let $L$ be the Galois closure of $K$ over $\boldsymbol{Q}$, and put

$$G = \mathrm{Gal}(L/\boldsymbol{Q}), \quad H = \mathrm{Gal}(L/K), \quad d = [K\colon \boldsymbol{Q}]/2 .$$

We can canonically identify the embeddings of $K$ into $\boldsymbol{C}$ with the cosets $H\backslash G$ ($G$ acts on $K$ on the right). We denote the complex conjugation by $\rho$, which belongs to the center of $G$.

Let $S$ be a subset of $H\backslash G$ such that

$$H\backslash G = S \cup S\rho \quad \text{(disjoint union)}.$$

The pair $(K, S)$ or the triple $(G, H, S)$ is called a *CM-type*. Put

$$\tilde{S} = \{g \in G \mid Hg \in S\} .$$

We say that a CM-type $(K, S)$ is *simple* if

$$H = \{g \in G \mid g\tilde{S} = \tilde{S}\} .$$

Let

$$H' = \{g \in G \mid \tilde{S}g = \tilde{S}\} ,$$

and $K'$ be the corresponding CM-field. Let $S'$ be the subset of $H'\backslash G$ which is induced by the inverses of the elements of $\tilde{S}$. Then the pair $(K', S')$ is also a CM-type, which is called the *dual* of $(K, S)$.

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