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COMPLETELY OPERATOR-SELFDECOMPOSABLE DISTRIBUTIONS AND OPERATOR-STABLE DISTRIBUTIONS

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§1. Introduction

Urbanik introduces in [16] and [17] the classes L_m and L_∞ of distributions on R^1 and finds relations with stable distributions. Kumar-Schreiber [6] and Thu [14] extend some of the results to distributions on Banach spaces. Sato [7] gives alternative definitions of the classes L_m and L_∞ and studies their properties on R^d. Earlier Sharpe [12] began investigation of operator-stable distributions and, subsequently, Urbanik [15] considered the operator version of the class L on R^d. Jurek [3] generalizes some of Sato's results [7] to the classes associated with one-parameter groups of linear operators in Banach spaces. Analogues of Urbanik's classes L_m (or L_∞) in the operator case are called multiply (or completely) operator-selfdecomposable. They are studied in relation with processes of Ornstein-Uhlenbeck type or with stochastic integrals based on processes with homogeneous independent increments (Wolfe [18], [19], Jurek-Vervaat [5], Jurek [2], [4], and Sato-Yamazato [9], [10]). The purpose of the present paper is to continue the preceding papers, to give explicit characterizations of completely operator-selfdecomposable distributions and operator-stable distributions on R^d, and to establish relations between the two classes. For this purpose we explore the connection of the structures of these classes with the Jordan decomposition of a basic operator Q.

Let $\mathscr{P}(R^d)$ be the class of probability distributions on R^d, and $M_+(R^d)$ be the class of linear operators on R^d all of whose eigenvalues have positive real parts. Let

$$t^Q = \sum_{n=0}^{\infty} (n!)^{-1}(\log t)^n Q^n \qquad \text{for } t > 0 .$$

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