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# ON SOLUTIONS OF VARIATIONAL INEQUALITIES CONSTRAINED ON A SUBSET OF POSITIVE CAPACITY

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**1.** Let $\Omega$ be a bounded domain of $R^n$ with boundary $\partial\Omega$ and let $E$ be a compact subset of $\Omega$. We assume that both $\partial\Omega$ and $E$ have positive capacity. The norm and the inner product in $L^2(\Omega)$ are simply denoted by $\|\ \ \|$ and $(\ ,\ )$ respectively. We define $\|u\|_1 = \|\nabla u\|$. The completion of $C_0^1(\Omega)$ with respect to the norm $\|u\|_1$ is denoted by $H_0^1(\Omega)$, where $C_0^1(\Omega)$ is the set of all functions in $C^1(\Omega)$ with compact support in $\Omega$. The inner product of $H_0^1(\Omega)$ is written with $(\ ,\ )_1$. We denote by $H^{-1}(\Omega)$ the dual space of $H_0^1(\Omega)$ and by $\|\ \ \|_{-1}$ its norm.

Let $K$ be a closed convex set in $H_0^1(\Omega)$ such that each element of $K$ is constraind only on $E$, that is, if $v \in H_0^1(\Omega)$ and $v = u$ on $E$ for some $u \in K$, then $v \in K$[1]. It is known that for any given $g \in H^{-1}(\Omega)$, there is a unique solution $u \in K$ of

$$(1.1) \qquad (u, v - u)_1 \geqq (g, v - u) \qquad \text{for all } v \in K$$

and if $g$ is besides in $L^2_{\text{loc}}(\Omega - E)$, the weak second derivatives $\partial^2 u$ also are there, though $\partial^2 u$ are distributions over $\Omega$.

In particular, when $g = 0$ and $K$ equals to

$$K_1 = \{v \in H_0^1(\Omega);\ v \geqq \psi \text{ on } E \text{ in the sense of } H_0^1(\Omega)\}^{2)}$$

for a given function $\psi \in C^1(\bar{\Omega})$, H. Lewy and G. Stampacchia [11] showed that the solution $u$ of (1.1) is in $C^0(\bar{\Omega})$ under certain assumptions on $E$ and $\partial\Omega$, for instance, $\Omega$ is a disk and $E$ is a segment. Their method is potential-theoretic.

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1) More precisely, there are two approximating sequences $\{u_j\}, \{v_j\} \subset H_0^1(\Omega) \cap C^0(\Omega)$ such that $u_j \to u$, $v_j \to v$ in $H_0^1(\Omega)$ and $u_j = v_j$ on $E$. Thus $v = u$ on $E$ except for a set of capacity zero.

2) The precise definition of $K_1$ is referred to [11].