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ON DEGREES AND GENERA OF CURVES ON SMOOTH QUARTIC SURFACES IN P^3

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Our result is motivated by the results [GP] of Gruson and Peskin on characterization of the pair of degree d and genus g of a non-singular curve in P^3. In the last step, they construct the required curve C on a singular quartic surface when $g \leqq (d-1)^2/8$. Here we consider curves on smooth quartic surfaces.

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THEOREM 1. *Let k be an algebraically closed field of characteristic* 0 *and $d > 0$ and $g \geqq 0$ be integers. Then there is a non-singular curve C of degree d and genus g on a non-singular quartic surface X in P^3 if and only if* (1) $g = d^2/8 + 1$, *or* (2) $g < d^2/8$ and $(d, g) \neq (5, 3)$.

Remark 2. Under the notation of Theorem 1, $g = d^2/8 + 1$ if and only if C is a complete intersection of X and a hypersurface of degree $d/4$, which will be proved in the proof below.

Proof of the only-if-part ($\Rightarrow$) *of Theorem* 1. Let $H = \mathcal{O}_X(1)$. Since $(H \cdot H) > 0$, one has

$$(C \cdot H)^2 - (H \cdot H) \cdot (C \cdot C) = d^2 - 8(g-1) \geqq 0\,,$$

by Hodge index theorem, because X is a $K3$ surface and $K_C = \mathcal{O}_C(C)$. One has $d^2 \equiv 0, 1, 4, 1 \pmod 8$ according as $d \equiv 0, 1, 2, 3 \pmod 4$. If $d^2 - 8(g-1) = 0$ then the classes of $\mathcal{O}_X(C)$ and $\mathcal{O}_X(H)$ are proportional. Since X is a $K3$ surface and $(H \cdot H) = 4$, $\operatorname{Pic} X$ is torsion-free and H is not divisible, whence $\mathcal{O}_X(C)$ is a multiple of $\mathcal{O}_X(H)$, which implies that C is a complete intersection of X and a hypersurface of degree $d/4$. It

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