L. Ein and I. Sols
Nagoya Math. J.
Vol. 96 (1984), 11-22

STABLE VECTOR BUNDLES ON QUADRIC HYPERSURFACES

L. EIN AND I. SOLS

§0.

Barth, Hulek and Maruyama have showed that the moduli of stable rank 2 vector bundles on $\boldsymbol{P}^2$ are nonsingular rational varieties. There are also many examples of stable rank 2 vector bundles on $\boldsymbol{P}^3$. On the other hand, there is essentially only one example of rank 2 bundles on $\boldsymbol{P}^4$, which is constructed by Horrocks and Mumford. We hope the study of rank 2 bundles on hypersurfaces in $\boldsymbol{P}^4$ may give more insight to the study of vector bundles on $\boldsymbol{P}^4$. In this paper, we establish some general properties of stable rank 2 bundles on quadric hypersurfaces. We show the restriction theorem (1.4), (1.6), the existence of the spectrum (2.2), and the vanishing theorem (2.4), are also true for the stable rank 2 reflexive sheaves on quadric hypersurfaces just as in the case when the base variety is $\boldsymbol{P}^n$. Though the methods to prove such results are similar to those we use for projective spaces, there are some technical difficulties. We should also mention that we shall always assume the base field is characteristic 0 and algebraically closed, and we shall use the definition of stability introduced by Mumford and Takemoto.

§1.

We let Q_n be a nonsingular quadric hypersurface in $\boldsymbol{P}^{n+1}$. There is the following incidence correspondence:

$$\begin{array}{ccc} X & \xrightarrow{q} & Y \subseteq G(1,4) \\ {\scriptstyle p}\downarrow & & \\ Q_3 & & \end{array} \tag{1.A}$$

where Y is the subvariety in $G(1, 4)$, which corresponds to the set of lines in Q_3. X is the corresponding universal $\boldsymbol{P}^1$-bundle on Y. It is not hard to check that X a conic bundle over Q_3.

Received May 9, 1983.