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THE HODGE COHOMOLOGY AND CUBIC EQUIVALENCES

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In 1969, Mumford [8] proved that, for a complete non-singular algebraic surface F over the complex number field C, the dimension of the Chow group of zero-cycles on F is infinite if the geometric genus of F is positive. To this end, he defined a regular 2-form η_f on a non-singular variety S for a regular 2-form η on F and for a morphism $f: S \to S^n F$, where $S^n F$ is the n-th symmetric product of F, and he showed that η_f vanishes if all 0-cycles $f(s)$, $s \in S$, are rationally equivalent. Roitman [9] later generalized this to a higher dimensional smooth projective variety V. For $\omega \in H^0(V, \Omega_V^q)$, he has defined $\omega_n \in H^0(S^n V, \Omega^q)$ and proved that ω_n has the following property: if $f, g: S \to S^n V$ are morphisms such that the zero cycles $f(s)$ and $g(s)$ are rationally equivalent for every $s \in S$, then $f^*\omega_n = g^*\omega_n$. We may say this property, roughly, like this: $f^*\omega_n$ cannot distinguish the rational equivalence relation. The rational equivalence is the finest equivalence relation among the adequate equivalence relations (cf. [12]). We can therefore pose the problem: which equivalence relation can $f^*\omega_n$ distinguish and which one can $f^*\omega_n$ not?

On the other hand, Samuel has defined the cubic equivalences in [12]. Consider an algebraic family of cycles on a smooth projective variety V over an algebraically closed field k, parametrized by a smooth variety S. We can regard this family as a "function" on S with values in the set of cycles on V. A cycle algebraically equivalent to zero can be considered as the difference of values at two points for an appropriate "function" on a smooth projective curve. We shall assume that the parameter space S is a product of two curves $C_1 \times C_2$. Then we can define a difference of the second order: take two points $a_i^{(0)}$ and $a_i^{(1)}$ on C_i, $i = 1, 2$, respectively, and form a difference of values at $(a_1^{(0)}, a_2^{(0)})$ and $(a_1^{(1)}, a_2^{(0)})$. We also form a difference between $(a_1^{(0)}, a_2^{(1)})$ and $(a_1^{(1)}, a_2^{(1)})$. The difference of the second order is the difference of these two differences.

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