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ON THE HOLONOMY LIE ALGEBRA AND THE NILPOTENT COMPLETION OF THE FUNDAMENTAL GROUP OF THE COMPLEMENT OF HYPERSURFACES

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§1. Introduction

The purpose of this paper is to establish the following isomorphism of Lie algebras.

MAIN THEOREM. *Let X be the complement of a hypersurface S in the complex projective space $\boldsymbol{P}^N$. Then the tower of nilpotent complex Lie algebras associated with the fundamental group $\pi_1(X, *)$ and the holonomy Lie algebra $\mathfrak{g}_S$ attached to S are isomorphic. In particular, if S is the union of hyperplanes $\bigcup_{j=1}^{m+1} S_j$ in $\boldsymbol{P}^N$, the nilpotent completion of $\pi_1(X, *)$ is isomorphic to the nilpotent completion of*

$$\mathrm{Lib}\,(X_1, X_2, \cdots, X_{m+1})/\mathscr{A}$$

where we denote by $\mathrm{Lib}\,(X_1, X_2, \cdots, X_{m+1})$ *a free Lie algebra generated by* $X_1, X_2, \cdots, X_{m+1}$ *over* $\boldsymbol{C}$, *and* $\mathscr{A}$ *is the homogeneous ideal generated by the following elements*

I) $\sum_{j=1}^{m+1} X_j$,

II) $[X_{\nu_j}, X_{\nu_1} + \cdots + X_{\nu_p}]$, $\quad 1 \leqq j \leqq p$

where the hyperplanes $S_{\nu_1}, \cdots, S_{\nu_p}$ *satisfy* $H \cap S_{\nu_1} \cap \cdots \cap S_{\nu_p} \neq \phi$ *for a generic plane* H *and* $H \cap S_{\nu_1} \cap \cdots \cap S_{\nu_p} \cap S_k = \phi$ *if* $k \notin \{\nu_1, \cdots, \nu_p\}$.

For a smooth manifold we have a surjective homomorphism from the tower of the nilpotent completion of the holonomy Lie algebra to the tower of the nilpotent complex Lie algebras associated with the fundamental group (cf. [C]). Our main theorem guarantees that this map is an isomorphism in the case of the complement of hypersurfaces (cf. [A] Theorem 2).

In Section 2 we review the notion of holonomy Lie algebras and

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