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Nagoya Math. J.
Vol. 91 (1983), 163–172

THREEFOLDS WITH NEGATIVE KODAIRA DIMENSION AND POSITIVE IRREGULARITY

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§ 0. Introduction

The purpose of this paper is to study threefolds X, with negative Kodaira dimension $\kappa(X)$ and positive irregularity $q(X)$, defined over the complex field $\boldsymbol{C}$.

In Section 1 we recall some definitions and preliminary results. The main statements are contained in Section 2. We prove the following:

I) Assume the Euler-Poincaré characteristic $\chi(\mathcal{O}_X)$ is positive. Then X is birationally equivalent to a conic bundle on a surface S such that $\kappa(S) \geqslant 0$.

II) Suppose $\chi(\mathcal{O}_X) < 0$. Then there exist a projective nonsingular curve C of positive genus and a morphism $X \to C$ such that the general fibre is a rational surface.

Statement I) also follows by combining some results due to T. Mabuchi and K. Ueno. Precisely, X is uniruled whenever $q(X) > 0$, as pointed out by K. Ueno in [U2]. Using this fact, then the assert can be obtained from a more general result contained in [M], that requires a rather hard and lengthy proof. Our argument is more direct and it does not use the uniruledness of X.

Statement II) gives also the converse of another result due to T. Mabuchi (see [M], 2. 3. 2.).

In case $\chi(\mathcal{O}_X) = 0$ then X falls into item I) or II) according to whether $H^0(X, S^{12}(\Omega^2_X))$ has positive or zero dimension.

Finally, in Section 3 a more explicit description of threefolds belonging to family II) is given by using the Enriques-Iskovskih classification of minimal rational surfaces (see [I], Theorem 1). Precisely, we show that there exists a birational minimal model $\tilde{X}$ of X such that:

a) $\tilde{X} = C \times \boldsymbol{P}^2$, or

Received March 29, 1982.