N. Toda
Nagoya Math. J.
Vol. 91 (1983), 37-47

ON THE FUNDAMENTAL INEQUALITY FOR DEGENERATE SYSTEMS OF ENTIRE FUNCTIONS

Dedicated to Professor H. Ohtsuka on the occasion of his sixtieth birthday

NOBUSHIGE TODA

§1. Introduction

Let $f = (f_0, f_1, \cdots, f_n)$ $(n \geqq 1)$ be a transcendental system in $|z| < \infty$. That is, $f_0, f_1, \cdots, f_n$ are entire functions without common zeros and the characteristic function of f defined by H. Cartan ([1]):

$$T(r, f) = \frac{1}{2\pi} \int_0^{2\pi} U(re^{i\theta}) d\theta - U(0) ,$$

where

$$U(z) = \max_{0 \leqq j \leqq n} \log |f_j(z)| ,$$

satisfies the condition

$$\lim_{r \to \infty} \frac{T(r, f)}{\log r} = \infty .$$

Let X be a set of linear combinations $(\not\equiv 0)$ of $f_0, f_1, \cdots, f_n$ with coefficients in C in general position; that is, for any $n + 1$ elements

$$a_{0j} f_0 + a_{1j} f_1 + \cdots + a_{nj} f_n \qquad (j = 1, \cdots, n + 1)$$

in X, $n + 1$ vectors $(a_{0j}, a_{1j}, \cdots, a_{nj})$ are linearly independent, and

$$\lambda = \dim \{(c_0, c_1, \cdots, c_n) \in C^{n+1};\ c_0 f_0 + c_1 f_1 + \cdots + c_n f_n = 0\} .$$

It is clear that $0 \leqq \lambda \leqq n - 1$. We note that, for any $n + 1$ elements $F_0, F_1, \cdots, F_n$ in X,

$$\dim \{(c_0, c_1, \cdots, c_n) \in C^{n+1};\ c_0 F_0 + c_1 F_1 + \cdots + c_n F_n = 0\}$$

is also equal to λ. We say that the system f is degenerate when $\lambda > 0$.

About fifty years ago, H. Cartan ([1]) proved

Received September 18, 1981.