M. Brodmann
Nagoya Math. J.
Vol. 88 (1982), 1-16

REES RINGS AND FORM RINGS OF ALMOST COMPLETE INTERSECTIONS

MARKUS BRODMANN

1. Introduction

Recently different authors have studied the conormal modules I/I^2 of almost complete intersections in local Gorenstein rings (c. t. Aoyama [1], Herzog [8], Kunz [13], Matsuoka [16]). An essential tool in these papers is the theory of canonical modules and the fact that these modules are easy to handle in the case of almost complete intersections.

In this paper we rely on the idea that almost complete intersections should not behave very different from complete intersection and that therefore similar arguments must be possible to study Rees rings and form rings. The basic similarity is that almost complete intersections admit superregular sequences of length height (I). This is shown in section 2.

Our aim is to study to which extend two basic results of Rees [19] and Valla [22] extend from complete intersections to almost complete intersections. Rees' result states the well known fact that $\mathrm{Gr}_R(I)$ is a polynomial algebra if R is CM and I a complete intersection. Valla's Result claims that $R]I^n[$, $R\rangle I^n\langle$ and $\mathrm{Gr}_R(I^n)$ are CM under the same hypotheses for all $n>0$. The way Valla's result ought to be generalized is clear: By calculating the lengths of maximal regular sequences in the homogeneous maximal ideals of the above rings in terms of depth (R/I) and $\dim(R)$. This will be done in section 4.

How to generalize Rees' result is not evident. A reasonable way to do this may be a study of the geometry of the conormal cone $\mathrm{Spec}(\mathrm{Gr}_R(I))$. An attempt to this is given in section 6, where we consider the relations between the irreducible components of the conormal cone of I and

Received August 1, 1979.
Revised February 23, 1981.
(1) Supported by "Schweizerischer Nationalfonds zur Forderung der wissenschaftlichen Forschung"