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CURVATURE, GEODESICS AND THE BROWNIAN MOTION ON A RIEMANNIAN MANIFOLD I

RECURRENCE PROPERTIES

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§ 0. Introduction

Let M be an n-dimensional, complete, connected and locally compact Riemannian manifold and g be its metric. Denote by Δ_M the Laplacian on M.

The Brownian motion on the Riemannian manifold M is defined to be the unique minimal diffusion process $(X_t, \zeta, P_x, x \in M)$ associated to the Laplacian Δ_M where ζ is the explosion time i.e. if $\zeta(\omega) < +\infty$, $\lim_{t \to \zeta} X_t(\omega) = \infty$. It should be remarked that $\zeta = +\infty$ a.s. if M is compact. See Mckean [7], page 90.

The Brownian motion X on M is said to be recurrent if for every open subset U of M

$$P_x\{\omega | X_t(\omega) \in U \text{ for some } t > 0\} = 1 \text{ on } M\,;$$

otherwise it will be called transient.

It has been known that the Brownian motion on a compact Riemannian manifold is recurrent. See for example Mckean [7] page 99. In this paper we shall restrict our consideration to non compact case and clarify the relation between the recurrent and transient properties of the process X and geodesics, curvature of M.

In the first section we shall summarize recurrence and transience of the Brownian motion on a rotationally symmetric Riemannian manifold (See Section 1 for the precise definition). Section 2 is devoted to the discussion for the general case. Some examples will be shown in the last section.

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