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Nagoya Math. J.
Vol. 86 (1982), 229-248

MODULAR REPRESENTATIONS OF ABELIAN GROUPS WITH REGULAR RINGS OF INVARIANTS

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§1. Introduction

Let k be a field of characteristic p and G a finite subgroup of $GL(V)$ where V is a finite dimensional vector space over k. Then G acts naturally on the symmetric algebra $k[V]$ of V. We denote by $k[V]^G$ the subring of $k[V]$ consisting of all invariant polynomials under this action of G. The following theorem is well known.

THEOREM 1.1 (Chevalley-Serre, cf. [1, 2, 3]). *Assume that $p = 0$ or $(|G|, p) = 1$. Then $k[V]^G$ is a polynomial ring if and only if G is generated by pseudo-reflections in $GL(V)$.*

Now we suppose that $|G|$ is divisible by the characteristic $p(> 0)$. Serre gave a necessary condition for $k[V]^G$ to be a polynomial ring as follows.

THEOREM 1.2 (Serre, cf. [1, 3]). *If $k[V]^G$ is a polynomial ring, then G is generated by pseudo-reflections in $GL(V)$.*

But the ring $k[V]^G$ of invariants is not always a polynomial ring, when G is generated by pseudo-reflections in $GL(V)$ (cf. [1, 3]).

In this paper we shall completely determine abelian groups G such that $\boldsymbol{F}_p[V]^G$ are polynomial rings ($\boldsymbol{F}_p$ is the field of p elements). Our main result is

THEOREM 1.3. *Let V be a vector space over $\boldsymbol{F}_p$ and G an abelian group generated by pseudo-reflections in $GL(V)$. Let G_p denote the p-part of G and assume that $G_p \neq \{1\}$. Then the following statements on G are equivalent:*

(1) *$\boldsymbol{F}_p[V]^G$ is a polynomial ring.*

(2) *The natural $\boldsymbol{F}_p G_p$-module V defines a couple (V, G_p) which decomposes to one dimensional subcouples (for definitions, see § 2).*

Received June 19, 1980.