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THE INDEX OF ELLIPTIC OPERATORS OVER V-MANIFOLDS

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Introduction

Let M be a compact smooth manifold and let G be a finite group acting smoothly on M. Let E and F be smooth G-equivariant complex vector bundles over M and let $P: \mathscr{C}^\infty(M; E) \to \mathscr{C}^\infty(M; F)$ be a G-invariant elliptic pseudo-differential operator. Then the kernel and the cokernel of the operator P are finite-dimensional representations of G. The difference of the characters of these representations is an element of the representation ring $R(G)$ of G and is called the G-index of the operator P.

$$(1) \qquad \operatorname{ind} P = \operatorname{char}[\text{kernel } P] - \operatorname{char}[\text{cokernel } P] .$$

It is well-known that the G-index $\operatorname{ind} P \in R(G)$ depends only on the homotopy class of the elliptic operator and, as Atiyah and Singer showed in [2], $\operatorname{ind} P$ is determined by the stable equivalence class $[\sigma(P)] \in K_G(\tau M)$ of the principal symbol $\sigma(P)$ viewed as the difference bundle over the tangent bundle τM. The Atiyah-Singer index theorem asserts that the value $(\operatorname{ind} P)(g)$ is expressed by the evaluation of a certain characteristic class over the tangent bundle $\tau(M^g)$ of the fixed point set M^g.

$$(2) \qquad (\operatorname{ind} P)(g) = (-1)^{\dim M^g} \langle \operatorname{ch}^g [\sigma(P)] \mathscr{I}^g(M),\ [\tau(M^g)] \rangle .$$

Here $\operatorname{ch}^g [\sigma(P)]$ is a class in the compactly supported cohomology group $H_c^*(\tau(M^g); C)$ expressed in the characteristic classes of the complex eigenvector bundles by the action of g on the stable vector bundle $[\sigma(P)|_{\tau(M^g)}]$. $\mathscr{I}^g(M)$ is a class in $H^*(M^g; C)$ expressed in the characteristic classes of the real and complex eigenvector bundles by the action of g on the real vector bundle $\tau M|_{M^g}$. We call these classes over the fixed point set as the residual characteristic classes.

Next we consider the index of the operator $P^G: \mathscr{C}^\infty(M; E)^G \to \mathscr{C}^\infty(M; F)^G$

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