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BOUNDARY BEHAVIOR OF POSITIVE HARMONIC FUNCTIONS IN BALLS OF R^n

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§ 1. Introduction

Let R^n be the real n-dimensional euclidean space. Elements of R^n are denoted by $x = (x_1, \cdots, x_n)$, and $\|x\|$ denotes the euclidean norm of x. The open ball $B(x, r)$ with center x and radius r is defined by

$$B(x, r) := \{y: \|y - x\| < r\}$$

and the sphere $S(x, r)$ is defined by

$$S(x, r) := \{y: \|y - x\| = r\} .$$

In particular $B := B(0, 1)$ is the unit ball and $S := S(0, 1)$ is the unit sphere.

Let u be a positive harmonic function in B. Then by the Herglotz Theorem ([2], p. 29) there exists a positive Borel measure μ on the unit sphere S such that

$$u(z) = \frac{1}{\sigma_n} \int_S P(z, x) d\mu(x) \qquad (x \in S)$$

holds for all $z \in B$. $P(z, x)$ is the Poisson kernel for B defined by

$$P(z, x) := \frac{1 - \|z\|^2}{\|z - x\|^n}$$

and σ_n is the surface area of S.

Now the question arises of the relationship between the limiting behavior of $u(z)$ as z approaches a boundary point and the measure μ on the boundary. To study this question we define the open polar cap $J(a, r)$ having center $a \in S$ and radius r by

$$J(a, r) := \{x \in S: \|x - a\| < r\}$$

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