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LEMMA ON LOGARITHMIC DERIVATIVES AND HOLOMORPHIC CURVES IN ALGEBRAIC VARIETIES[1)]

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Nevanlinna's lemma on logarithmic derivatives played an essential role in the proof of the second main theorem for meromorphic functions on the complex plane C (cf., e.g., [17]). In [19, Lemma 2.3] it was generalized for entire holomorphic curves $f\colon C \to M$ in a compact complex manifold M (Lemma 2.3 in [19] is still valid for non-Kähler M). Here we call, in general, a holomorphic mapping from a domain of C or a Riemann surface into M a holomorphic curve in M, and sometimes use it in the sense of its image if no confusion occurs. Applying the above generalized lemma on logarithmic derivatives to holomorphic curves $f\colon C \to V$ in a complex projective algebraic smooth variety V and making use of Ochiai [22, Theorem A], we had an inequality of the second main theorem type for f and divisors on V (see [19, Main Theorem] and [20]). Other generalizations of Nevanlinna's lemma on logarithmic derivatives were obtained by Nevanlinna [16], Griffiths-King [10, § 9] and Vitter [23].

In this paper we first deal with holomorphic curves $f\colon \Delta^* \to M$ from the punctured disc $\Delta^* = \{|z| \geqq 1\}$ with center at the infinity ∞ of the Riemann sphere into a compact Kähler manifold M. Our first aim is to prove the following lemma on logarithmic derivatives which is a generalization of Nevanlinna [16, III, p. 370] and will play a crucial role in §§ 3 and 4 (see § 1 as to the notation):

MAIN LEMMA (2.2). *Let $f\colon \Delta^* \to M$ be a holomorphic curve in M, $\omega \in H^0(M, \mathfrak{A}^1_M)$ a d-closed meromorphic 1-form with logarithmic poles and put $f^*\omega = \zeta(z)dz$. Then we have*

$$m(r, \zeta) \leqq O(\log^+ T_f(r)) + O(\log r)$$

as $r \to \infty$ except for $r \in E$, where E is a subset of $[1, \infty)$ with finite linear

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