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# RIEMANNIAN HOMOGENEOUS FOLIATIONS WITHOUT HOLONOMY

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## § 1. Introduction

Let $M$ be a compact connected $C^\infty$ manifold with a smooth Riemannian foliation $\mathscr{F}$. That is, $(M, \mathscr{F})$ admits a bundle-like metric in the sense of [7]. In [4] it is shown that if all leaves of $\mathscr{F}$ are closed without holonomy, then the space of leaves $M/\mathscr{F}$ of the foliation is a manifold and the natural projection $M \to M/\mathscr{F}$ is a locally trivial fibre space. In the present work we show that for certain of the Riemannian homogeneous foliations, holonomy is the only obstruction to the foliation being a fibration.

Let $G/K$ be a simply connected, even dimensional, positively curved Riemannian homogeneous space of a compact connected Lie group $G$ and let $\mathscr{F}$ be a homogeneous $G/K$-foliation of a compact connected manifold $M$. For example, $\mathscr{F}$ is a codimension $2q$ elliptic (i.e., homogeneous $SO(2q+1)/SO(2q) \cong S^{2q}$-) foliation. Then $\mathscr{F}$ is cohomologically a fibration in the sense that the base-like cohomology algebra of the foliated manifold $(M, \mathscr{F})$ is isomorphic to the de Rham cohomology algebra of $G/K$ [3]. The main result of this paper asserts that if $\mathscr{F}$ has no holonomy, then it is actually a fibration.

(1.1) THEOREM. *If $\mathscr{F}$ is without holonomy, then $M$ fibres over $G/K$ with the leaves of $\mathscr{F}$ as fibres.*

## § 2. Riemannian Homogeneous Foliations

In this section we prove (1.1) and use its proof to elucidate further properties of Riemannian homogeneous foliations.

Let $G/K$ be a connected homogeneous space on which $G$ acts effectively and let $\mathscr{F}$ be a homogeneous $G/K$-foliation of a connected manifold $M$. That is, $\mathscr{F}$ is defined by a $G/K$-cocycle $\{(U_\alpha, f_\alpha, \lambda_{g_{\alpha\beta}})\}_{\alpha,\beta\in A}$ where

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