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WEAKLY NORMAL VARIETIES: THE MULTICROSS SINGULARITY AND SOME VANISHING THEOREMS ON LOCAL COHOMOLOGY

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§ 1. Introduction

The foundations for this paper were developed in [5], "Seminormal rings and weakly normal varieties", where the historical framework and fundamental properties of weakly normal varieties were presented in detail. Here we devote our attention to the study of the multicross singularity and the role of local cohomology in the theory of weakly normal varieties.

In our earlier paper we presented an affine version of the multicross. In general we will say a point x on a variety X is a multicross if X at x is analytically isomorphic to X' at x' where X' is an affine multicross (see [5], Defn. 3.3). We then give a more geometric description in terms of the normalization of X. Using this characterization we are able to show that the set X_m of multicrosses on a weakly normal variety is open and dense. In fact its complement is a closed subvariety of codimension at least two so that every component of the singular locus (respectively, nonnormal locus) of codimension one must contain a multicross.

We also present the algebro-geometric analogue of the class of locally optimal spaces introduced in [1]. We call them the C-weakly normal varieties as they are characterized by the vanishing of the local cohomology with supports on $X\setminus X_m$. We apply a Hartogs theorem for weak normality to show that a C-weakly variety is indeed weakly normal. However, there are many weakly normal varieties that fail to be C-weakly normal (cf. Ex. 4.5).

A further stratification of the class of C-weakly normal varieties leads us to the E-weakly normal varieties. Briefly, these are the varieties whose

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