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Nagoya Math. J.
Vol. 83 (1981), 123–135

ON BUCHSBAUM RINGS OBTAINED BY GLUING

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§ 1. Introduction

Let A be a Noetherian local ring with maximal ideal m. In 1973 J. Barshay [1] showed that, if A is a Cohen-Macaulay ring, then so is the Rees algebra $R(q) = \bigoplus_{n \geq 0} q^n$ for every parameter ideal q of A (c.f. p. 93, Corollary). Recently the author and Y. Shimoda [5] have proved that the Rees algebra $R(q)$ is a Cohen-Macaulay ring for every parameter ideal q of A if and only if

$$(\sharp)\ A \text{ is a Buchsbaum ring and } H^i_m(A) = (0) \quad \text{for} \quad i \neq 1, \dim A.$$

(Here $H^i_m(A)$ denotes the i-th local cohomology module of A. See (2.1) for the definition of Buchsbaum rings.) Of course this is a complete answer to the question whether the converse of Barshay's result is true. Subsequently by the author [4] there was given a practical criterion for a local ring A to satisfy the condition ($\sharp$) (c.f. Theorem (1.1)). Applying it to the case where the ring A considered appears as the local ring at the irrelevant maximal ideal of an affine semigroup ring over a field, one may deduce a necessary and sufficient condition for A to be a Buchsbaum ring in terms of the corresponding semigroup. In this case A satisfies the condition ($\sharp$) if it is a Buchsbaum ring (c.f. [4], Theorem (3.1)).

The purpose of the present paper is to apply further this criterion to local rings obtained by gluing and to find a good deal of local rings for which the Rees algebras of parameter ideals are always Cohen-Macaulay. (See Section 2 for the detail on gluing.) The concept of Buchsbaum local rings has its root in an answer of W. Vogel [12] to a conjecture of D. A. Buchsbaum [2] (c.f. p. 228) and the basic properties of Buchsbaum local rings were given by J. Stückrad and W. Vogel (c.f. [8] and [9]). But, even though the theory of Buchsbaum singularities is now developing rapidly, it is required to establish their ubiquity together with a great deal of

Received August 29, 1979.