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COMPLETELY SUPERHARMONIC MEASURES FOR THE INFINITESIMAL GENERATOR A OF A DIFFUSION SEMI-GROUP AND POSITIVE EIGEN ELEMENTS OF A

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§ 1. Introduction

Let X be a locally compact Hausdorff space with countable basis. We denote by

$M(X)$ the topological vector space of all real Radon measures in X with the vague topology,

$M_K(X)$ the topological vector space of all real Radon measures in X whose supports are compact with the usual inductive limit topology.

Their subsets of all non-negative Radon measures are denoted by $M^+(X)$ and by $M_K^+(X)$, respectively.

In the paragraph 2, we shall prepare the terminology and the notation which we shall use in the sequel.

A continuous linear operator T from $M_K(X)$ into $M(X)$ is called a diffusion kernel on X if T is positive, i.e., $T\mu \in M^+(X)$ whenever $\mu \in M_K^+(X)$. A semi-group $(T_t)_{t\geqq 0}$ of diffusion kernels on X is called a diffusion semi-group if $T_0 = I$ (the identity) and if, for any $\mu \in M_K(X)$, the mapping $t \to T_t\mu$ is continuous in $M(X)$.

We consider the infinitesimal generator A of a transient and regular diffusion semi-group $(T_t)_{t\geqq 0}$ on X. A Radon measure $\mu \in M(X)$ is said to be A-superharmonic (resp. A-harmonic) if it satisfies $-A\mu \in M^+(X)$ (resp. $A\mu = 0$).

In the paragraph 3, we shall show that every positive A-superharmonic Radon measure is written uniquely as the sum of a V-potential of a non-negative Radon measure and a non-negative A-harmonic measure, where V is the Hunt diffusion kernel for $(T_t)_{t\geqq 0}$, i.e.,

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