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ON THE NORM CONTINUITY OF $\mathscr{S}'$-VALUED GAUSSIAN PROCESSES

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Summary

Let $\mathscr{S}$ be the Schwartz space of all rapidly decreasing functions on $\boldsymbol{R}^n$, $\mathscr{S}'$ be the topological dual space of $\mathscr{S}$ and for each positive integer p, $\mathscr{S}'_p$ be the space of all elements of $\mathscr{S}'$ which are continuous in the p-th norm defining the nuclear Fréchet topology of $\mathscr{S}$. The main purpose of the present paper is to show that if $\{X_t, t \in [0, +\infty)\}$ is an $\mathscr{S}'$-valued Gaussian process and for any fixed $\varphi \in \mathscr{S}$ the real Gaussian process $\{X_t(\varphi), t \in [0, +\infty)\}$ has a continuous version, then for any fixed $T > 0$ there is a positive integer p such that $\{X_t, t \in [0, T]\}$ has a version which is continuous in the norm topology of $\mathscr{S}'_p$.

§ 1. Introduction

Let E be a locally convex topological vector space, E' be the topological dual space of E and denote by $C(E', E)$ the smallest σ-algebra of subsets of E' that makes all functions $\{\langle x, \xi\rangle : \xi \in E\}$ measurable, where $\langle x, \xi\rangle$ is the canonical bilinear form on $E' \times E$. An E'-valued stochastic process is a collection $X = \{X_t, t \in [0, +\infty)\}$ of measurable maps X_t from a complete probability space $(\Omega, \mathscr{B}, P)$ into the measurable space $(E', C(E', E))$. Throughout this paper R_+, T_+ and N denote the half line $[0, +\infty)$, the closed interval $[0, T]$ and the set of all positive integers.

X is said to be *Gaussian* if the family of real random variables $\{\langle X_t, \xi\rangle : t \in R_+, \xi \in E\}$ forms a Gaussian system.

We shall study below sample path continuity of E'-valued Gaussian processes in case where E is a nuclear Fréchet space or a countable strict inductive limit of nuclear Fréchet spaces. In the following definitions we assume that E is one of such spaces. Then the Borel field of E' coincides with $C(E', E)$. If X is Gaussian the probability law μ_t of X_t which is

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