Y. Kanie
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SOME LIE ALGEBRAS OF VECTOR FIELDS AND THEIR DERIVATIONS CASE OF PARTIALLY CLASSICAL TYPE

YUKIHIRO KANIE

Introduction

Let $(M, \mathscr{F})$ be a smooth foliated manifold, and $\mathscr{T}(M, \mathscr{F})$ the Lie algebra of all leaf-tangent vector fields on M.

Assume that $(M, \mathscr{F})$ admits a partially classical structure τ, ω or θ (see § 4.1). Then we have natural Lie subalgebras $\mathscr{T}_\tau(M, \mathscr{F})$, $\mathscr{T}_{c\tau}(M, \mathscr{F})$, $\mathscr{T}_\omega(M, \mathscr{F})$, $\mathscr{T}_{c\omega}(M, \mathscr{F})$, $\mathscr{T}_\theta(M, \mathscr{F})$ of the Lie algebra $\mathscr{T}(M, \mathscr{F}) = \mathscr{T}_0(M, \mathscr{F})$ (see § 4.2). These Lie algebras including $\mathscr{T}(M, \mathscr{F})$ itself are called of partially classical type. Here we study the structures of those Lie algebras and their derivation algebras.

The derivation algebra of $\mathscr{T}(M, \mathscr{F})$ is naturally isomorphic to the Lie algebra $\mathscr{L}(M, \mathscr{F})$ of all locally foliation-preserving vector fields on M (see [4]). We get similarly natural Lie subalgebras $\mathscr{L}_\tau(M, \mathscr{F})$, $\mathscr{L}_{c\tau}(M, \mathscr{F})$, $\mathscr{L}_\omega(M, \mathscr{F})$, $\mathscr{L}_{c\omega}(M, \mathscr{F})$, $\mathscr{L}_\theta(M, \mathscr{F})$ of $\mathscr{L}(M, \mathscr{F}) = \mathscr{L}_0(M, \mathscr{F})$ (see § 4.2).

Our main results (announced in [11]) are

MAIN THEOREM. *Let M be a smooth $(p+q)$-dimensional manifold, and $\mathscr{F}$ a codimension q foliation on M. Assume that $(M, \mathscr{F})$ is equipped with a partially classical structure τ, ω or θ.*

(a) *Let $\sigma = 0$, $c\tau (p \neq 1)$, $c\omega$ or θ. Then*

$$H^1(\mathscr{L}_\sigma(M, \mathscr{F}); \mathscr{L}_\sigma(M, \mathscr{F})) = 0 ,$$
$$H^1(\mathscr{T}_\sigma(M, \mathscr{F}); \mathscr{T}_\sigma(M, \mathscr{F})) \cong \mathscr{L}_\sigma(M, \mathscr{F})/\mathscr{T}_\sigma(M, \mathscr{F}) .$$

(b) *Let $\sigma = \tau (p \neq 1)$ or ω. Then*

$$H^1(\mathscr{L}_\sigma(M, \mathscr{F}); \mathscr{L}_\sigma(M, \mathscr{F})) \cong \mathscr{L}_{c\sigma}(M, \mathscr{F})/\mathscr{L}_\sigma(M, \mathscr{F}) ,$$
$$H^1(\mathscr{T}_\sigma(M, \mathscr{F}); \mathscr{T}_\sigma(M, \mathscr{F})) \cong \mathscr{L}_{c\sigma}(M, \mathscr{F})/\mathscr{T}_\sigma(M, \mathscr{F}) .$$

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