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THE PRINCIPLE OF LIMITING ABSORPTION FOR UNIFORMLY PROPAGATIVE SYSTEMS WITH PERTURBATIONS OF LONG-RANGE CLASS

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§1. Introduction

The aim of this paper is to establish the principle of limiting absorption for uniformly propagative systems $\Lambda(x, D_x) = E(x)^{-1} \sum_{j=1}^{n} A_j D_j$, $D_j = -i\partial/\partial x_j$, with perturbations of long-range class, where the perturbation of long-range class, roughly speaking, means that $E(x)$ approaches to E_0, E_0 being the $N \times N$ identity matrix, as $|x| \to \infty$ with order $O(|x|^{-\delta})$, $0 < \delta \leqq 1$. (The more precise assumptions will be stated below and we require some additional assumptions on the derivatives of $E(x)$.) The spectral and scattering problem for uniformly propagative systems was first formulated by Wilcox [10]. Since then, the principle of limiting absorption has been proved by many authors ([5], [7], [8], [11] etc.). The perturbations discussed in their works belong to the short-range class with $\delta > 1$.

On the other hand, for the Schrödinger operators with long-range potentials, this principle has been already verified by many authors ([2], [3], [6] etc.). Especially, S. Agmon has extended their results to general elliptic operators of higher order, using the localization theory in the momentum space, ξ-space (lecture given at the Kyoto University, 1977).

In this paper, we also use the localization theory, so we owe much to Agmon's idea. However, his method cannot be directly applied to our problem. In particular, when the characteristic equation for the unperturbed system $\Lambda_0(D_x) = \sum_{j=1}^{n} A_j D_j$ has multiple roots, a few difficulties occur and we need some modifications.

1.1. Notations. We first list up the notations which will be used throughout this paper. (1) $\boldsymbol{R}_x^n$ and $\boldsymbol{R}_\xi^n$ denote the n-dimensional euclidean space with generic points $x = (x_1, \cdots, x_n)$ and $\xi = (\xi_1, \cdots, \xi_n)$, respectively.

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