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DEFORMATIONS AND EQUITOPOLOGICAL DEFORMATIONS OF STRONGLY PSEUDOCONVEX MANIFOLDS

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§1. Introduction

One of the main problems in complex analysis has been to determine when two open sets D_1, D_2 in C^n are biholomorphically equivalent. In [26] Poincaré studied perturbations of the unit ball B_2 in C^2 of a particular kind, and found necessary and sufficient conditions on a first order perturbation that the perturbed domain be biholomorphically equivalent to B_2. Recently Burns, Shnider and Wells [7] (cf. also Chern-Moser [9]) have studied the deformations of strongly pseudoconvex manifolds. They proved that there is no finite-dimensional deformation theory for M if one keeps track of the boundary.

In view of this, we have the following definition.

Definition. Let M and M' be two strongly pseudoconvex manifolds with A and A' as its maximal compact analytic set. M is said to be holomorphically equivalent to M' if there exist open neighborhoods U and U' of A and A' respectively and biholomorphic map $\varphi\colon U \to U'$ such that $\varphi(A) = A'$.

The natural question one can raise is to determine geometric conditions which imply that M and M' are holomorphically equivalent.

Let M be a strongly pseudoconvex manifold with a one-dimensional exceptional set A. Let Θ be the holomorphic tangent sheaf to M. The general Kodaira-Spencer [18] theory shows that $H^1(M, \Theta)$ corresponds to first order infinitesimal deformations of M and that $H^2(M, \Theta)$ represents the obstructions to formally extending deformations to higher order. $H^1(M, \Theta)$ is finite-dimensional since M is strongly pseudoconvex. $H^2(M, \Theta) = 0$ since A is one-dimensional. Because of the result of Burns, Schneider and Wells we mentioned above, given a deformation of M and a compact K in M,

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