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SEMINORMAL RINGS AND WEAKLY NORMAL VARIETIES

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Introduction

In the late sixties and early seventies the operation of weak normalization was formally introduced first in the case of analytic spaces and later in the abstract scheme setting (cf. [6] & [4]). The notion arose from a classification problem. An unfortunate phenomenon in this area occurs when one tries to parametrize algebraic objects associated with a space by an algebraic variety; the resulting variety is, in general, not uniquely determined and may, for example, depend on the choice of coordinates. Under certain conditions one does know that the normalization of the parameter variety is unique. The price one pays for passing to the normalization is that often this variety no longer parametrizes what it was intended to; one point on the original parameter variety may split into several in the normalization. This problem is avoided if one passes instead to the weak normalization of the original variety. One then obtains a variety homeomorphic to the original variety with universal mapping properties that guarantee uniqueness.

In recent years weakly normal complex spaces have been systematically studied by several people and many interesting results have been obtained. On a complex space X define the sheaf of c-holomorphic functions $\mathcal{O}_X^c$ on X as follows. For an open subset U of X let $\Gamma(U, \mathcal{O}_X^c)$ consist of all continuous complex valued functions on U which are holomorphic at the regular points of U. X is weakly normal if $\mathcal{O}_X = \mathcal{O}_X^c$ (where $\mathcal{O}_X$ denotes the sheaf of holomorphic functions on X).

In [2] a generic type singularity called a multicross was defined and was shown to be what most frequently occurs in weakly normal spaces. More precisely, the complement of the multicrosses is an analytic subset of codimension at least two. One has a Hartogs theorem for weak normality (cf. [5] & [2] for a refinement) and an Oka theorem which com-

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