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TWO ALGEBRAIC DEFORMATIONS OF A $K3$ SURFACE

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Introduction

Let X be a nonsingular algebraic $K3$ surface carrying a nonsingular hyperelliptic curve of genus 3 and no rational curves. Our purpose is to study two algebraic deformations of X, viz. one specialization and one generalization. We assume the characteristic $\neq 2$. The generalization of X is a nonsingular quartic surface Q in $\boldsymbol{P}^3$: we wish to show in § 1 that there is an irreducible algebraic family of surfaces over the affine line, in which X is a member and in which Q is a general member. The specialization of X is a surface Y having a birational model which is a ramified double cover of a quadric cone in $\boldsymbol{P}^3$. It has been observed [19] that such a specialization exists; what we propose to show is that there are two different ways to get to it, that is there are two non-isomorphic irreducible algebraic families $\mathscr{S}$, $\mathscr{S}^*$ of surfaces over the affine line, each having a surface Y as a member and having X as a general member. In fact it is shown in § 3 that the "elementary operation" (cf. § 2) in the known family $\mathscr{S}$, along a single, nonextending nodal curve R on Y (i.e. R is a non-singular rational curve with self-intersection -2 on Y), exists algebraically—it is always defined analytically, [3], [9]—and defines a birational transformation η of $\mathscr{S}$. The image $\mathscr{S}^* = \eta(\mathscr{S})$ is an algebraic family over the affine line, not isomorphic to $\mathscr{S}$ but having the same members. We remark that, while $\mathscr{S}$ and $\mathscr{S}^*$ can be regarded separately as families of polarized surfaces, in the sense of [13], the birational correspondence η between them does not respect any structure of polarization of general members; hence there is no conflict with Theorem 2 in [15], even though η induces an isomorphism between general members X but the graph of η does not specialize to the graph of an isomorphism between special members Y. To establish the existence of η we apply a theorem

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