S. Mukai
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DUALITY BETWEEN $D(X)$ AND $D(\hat{X})$ WITH ITS APPLICATION TO PICARD SHEAVES

SHIGERU MUKAI

Introduction

As is well known, for a real vector space V, the Fourier transformation

$$\hat{f}(\alpha) = \int_V f(v)e^{2\pi i\langle v,\alpha\rangle}dv \qquad \alpha \in V^\vee$$

gives an isometry between $L^2(V)$ and $L^2(V^\vee)$, where $V^\vee$ is the dual vector space of V and $\langle\ ,\ \rangle\colon V \times V^\vee \to \boldsymbol{R}$ is the canonical pairing.

In this article, we shall show that an analogy holds for abelian varieties and sheaves of modules on them: Let X be an abelian variety, $\hat{X}$ its dual abelian variety and $\mathscr{P}$ the normalized Poincaré bundle on $X \times \hat{X}$. Define the functor $\hat{\mathscr{S}}$ of $\mathcal{O}_X$-modules M into the category of $\mathcal{O}_{\hat{X}}$-modules by

$$\hat{\mathscr{S}}(M) = \pi_{\hat{X},*}(\mathscr{P} \otimes \pi_X^* M)\ .$$

Then the derived functor $\boldsymbol{R}\hat{\mathscr{S}}$ of $\hat{\mathscr{S}}$ gives an equivalence of categories between two derived categories $\boldsymbol{D}(X)$ and $\boldsymbol{D}(\hat{X})$ (Theorem 2.2).

In § 3, we shall investigate the relations between our functor $\boldsymbol{R}\hat{\mathscr{S}}$ and other functors, translation, tensoring of line bundles, direct (inverse) image by an isogeny, etc. The result (3.14) that if X is principally polarized then $\boldsymbol{D}(X)$ has a natural action of $\mathrm{SL}(2, Z)$ seems to be significant.

In §§ 4 and 5, we shall apply the duality between $\boldsymbol{D}(X)$ and $\boldsymbol{D}(\hat{X})$ to the study of Picard sheaves. We shall compute the cohomology of Picard sheaves (Proposition 4.4), determine the moduli of deformations of them (Theorem 4.8) and give a characterization of them in the case of $\dim X = 2$ (Theorem 5.4). Other applications of the duality will be treated elsewhere.

After the original paper was written, the author learned by a letter from G. Kempf that Proposition 3.11 and some results in § 4 had also been proved independently by him.

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