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RINGS OF CONVERGENT POWER SERIES AND WEIERSTRASS PREPARATION THEOREM

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§0.

Let B be a B-ring with a nonarchimedean valuation $|\ |$, i.e., B is an integral domain satisfying the following conditions: (i) B is bounded ($|a| \leqq 1$ for every $a \in B$), (ii) the boundary $\partial(B) = \{a \in B; |a| = 1\}$ forms a multiplicative group. Let Z_+ denote the set of all nonnegative integers. Let $n \in Z_+$. Let $x_1, \cdots, x_n$ be n variables over B. We denote by $A_n = B\langle x_1, \cdots, x_n\rangle$ the set of all elements which can be written in the form

$$\sum_\nu a_\nu x^\nu ,$$

where $a_\nu \in B$ for all $\nu \in Z_+^n$ and $|a_\nu| \to 0$ as $\nu_1 + \cdots + \nu_n \to \infty$. We define a norm $\|\ \|$ on A_n: For $g = \sum a_\nu x^\nu \in A_n$, let $\|g\| = \max\{|a_\nu|\}$. Let m be the maximal ideal of B and $k = B/m$ be the residue field. Let τ be the canonical mapping of B onto k. Then τ can be extended to an epimorphism from A_n to a polynomial ring $k[x_1, \cdots, x_n]$ in the usual manner. We assume, throughout this paper, the B-ring B is complete. We shall identify $A_{n-1}\langle x_n\rangle$ with A_n so that each element g of A_n has an expression $\sum g_i x_n^i$, where $g_i \in A_{n-1}$ for all $i \in Z_+$ and $\|g_i\| \to 0$ as $i \to \infty$. For any $s \in Z_+$, let P_s denote the set of all polynomials of $A_{n-1}[X_n]$ of degree $< s$. One can see several properties on a B-ring in [2], [4].

In this paper, we shall prove Weierstrass Preparation Theorem for A_n. We shall obtain Weierstrass Form Theorem and Scherung Theorem for A_n also.

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