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ON SOME TYPES OF GEODESICS ON RIEMANNIAN MANIFOLDS

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§ 0. Introduction

For a given Riemannian manifold M and its submanifold N, one can find various types of geodesics on M starting from any point of N and ending in any point of N. For example, geodesics which start perpendicularly from N and end perpendicularly in N are treated by many mathematicians. K. Grove has stated a condition in a general case for the existence of such a geodesic ([4]), where he has used the method of the infinite dimensional critical point theory. This method is very useful for the study of geodesics and many geometricians have used it successfully. It has two aspects: one is an existence theory and the other is a quantitative theory, which one can find, for instance, in the excellent theory for closed geodesics of W. Klingenberg ([1], [7]) and so on.

On the other hand the works of K. Grove ([2], [4]) suggest us that this method is applicable to qualitative questions for geodesics. Here we shall study some types of geodesics from this point of view.

Let M be a complete Riemannian manifold and let N be a closed submanifold, then it seems very interesting to ask "Do there exist geodesics on M which start from N with one angle and end in N with the same angle?"

Unfortunately for our problem, we cannot use the infinite dimensional method directly, because it is difficult to find a satisfactory infinite dimensional manifold. Under some nice conditions, however, we can apply this method to our problem: that is to find a good isometry on M with respect to N.

We don't know whether our idea is extensible to a more general theory or not, but we are sure to give a little clue to solve a more general problem. And at the same time our work makes an example of the in-

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