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A CONTRIBUTION TO THE THEORY OF FORMAL MEROMORPHIC FUNCTIONS

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In my paper [F3] I more or less explicitly conjectured that if A is a complete local integral domain with maximal ideal $\mathfrak{m}$ and if $I = (t_1, \cdots, t_n)$ is an ideal in A with $n \leqslant \dim(A) - 2$, then $\operatorname{Spec}(A/I) - \mathfrak{m}$ is $G3$ in $\operatorname{Spec}(A) - \mathfrak{m}$. This will be proved in this paper.

Unfortunately the algebraisation-theorems for subsheaves in [F3] make great difficulties in this context, and I only can prove similar theorems, where I deal not with complete local rings but with a different category which will be explained in detail later. At least our theorems are sufficient to deal with projective varieties.

As corollary I obtain for example a strong generalisation of the $G3$-theorems in [S].

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§1. Notations

All rings are commutative, noetherian and have a unit. If A is such a ring, $\mathscr{I} \subseteq \mathfrak{a} \subseteq A$ two ideals, such that A is complete in the $\mathscr{I}$-adic topology, then $X := \operatorname{Spec}(A)$, $Y := \operatorname{Spec}(A/\mathscr{I}) \subseteq X$, $Z := \operatorname{Spec}(A/\mathfrak{a}) \subseteq Y$, $U := X - Z$, $\hat{X} := \operatorname{Spf}(A) =$ formal completion of X along Y and $\hat{U} :=$ formal completion of U along $U \cap Y$. If anywhere in this paper there appears a ring A with ideals $\mathscr{I}$ and $\mathfrak{a}$, these notations are used without further comment. If there occur other rings $A_1, A_2, \cdots$ with ideals $\mathscr{I}_1, \mathscr{I}_2, \cdots$ and $\mathfrak{a}_1, \mathfrak{a}_2, \cdots$, then $X_1, X_2, \cdots, U_1, U_2, \cdots$ etc. are defined according to these data in the same way as above.

If f moves in a system of generators for $\mathfrak{a}/\mathscr{I}$, then $\hat{U}$ can be glued together from the $\operatorname{Spf}(A_{(f)})$, where $A_{(f)}$ is the $\mathscr{I}$-adic completion of the ordinary localisation $A_{(f)}$. If M is a finitely generated A-module, then M induces a coherent sheaf on $\hat{X}$ and $\hat{U}$. Sheaves arising in this way are

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