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FIRST CHERN CLASS AND HOLOMORPHIC TENSOR FIELDS

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1. Introduction

Let M be an n-dimensional compact Kaehler manifold, TM its (holomorphic) tangent bundle and T^*M its cotangent bundle. Given a complex vector bundle E over M, we denote its m-th symmetric tensor power by S^mE and the space of holomorphic sections of E by $\Gamma(E)$. In [4] we have shown that $\Gamma(S^mTM) = 0$ (resp. $\Gamma(S^mT^*M) = 0$) if $c_1(M) \leq 0$ (resp. $c_1(M) \geq 0$) and if M is simply connected. (For the precise statement of a little stronger result, see [4]).

In this paper we consider more general tensor bundles. Our results may be summarized as follows:

THEOREM A. *Let M be a compact Kaehler manifold with $c_1(M) < 0$ (i.e., with ample canonical line bundle K_M). Let*

$$T^r_sM = \left(\overset{r}{\otimes}\, TM\right) \otimes \left(\overset{s}{\otimes}\, T^*M\right).$$

If $r > s$, then $\Gamma(T^r_sM) = 0$, i.e., there is no holomorphic tensor fields of contravariant degree r and covariant degree s.

The theorem above is an immediate consequence of a theorem of Bochner [7] and a recent result of Aubin [1] and Yau [8].

COROLLARY A.1. *Let m be as above. Let m be a non-negative integer and q a (possibly negative) integer. Then*

(1) $\Gamma(S^mTM \otimes K^q_M) = 0$ *for* $m - qn > 0$,

(2) $\Gamma(S^mT^*M \otimes K^q_M) = 0$ *for* $-m - qn > 0$.

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