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# AN EFFECTIVE METHOD OF COUNTING THE NUMBER OF LIMIT CYCLES

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## Introduction

We are interested in determining, after a finite number of procedures, the number and the approximate positions of limit cycles for a given system.

For instance, let

$$(*) \qquad \begin{cases} \dot{x} = f(x, y) \\ \dot{y} = g(x, y) \end{cases}$$

be a given autonomous system on an $(x, y)$-plane $\boldsymbol{R}^2$. Suppose that analytic expressions for the solutions of $(*)$ are not to be expected. Then in order to know the analytic properties of the solutions, we have to study the "pattern" described by the family of integral curves. For this purpose it is of basic importance to investigate the singular points (points such that $f(x, y) = g(x, y) = 0$) and the limit cycles (isolated closed integral curves), because the singular points and the limit cycles dominate the global pattern. The problem of investigating the singular points is the one of algebraic equations, while as to the problem of investigating the limit cycles, any effective, general method has not been known yet.

The purpose of the present paper is to give a method by which the number and the approximate positions of limit cycles can be determined. Indeed we shall show that

(i) our problem (i.e. determining the number and the approximate positions of limit cycles) can be reduced to the problem of finding approximate solutions of a partial differential equation (denoted by (E) below),

and as applications of our method to Liénard's equations we shall prove

(ii) the classical theorem concerning the generating circles of limit

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