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A GEOMETRIC CHARACTERIZATION OF C^n AND OPEN BALLS

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Introduction

The purpose of this paper is to give a result concerning the problem of geometric characterizations of the Euclidean n-space C^n and bounded domains. It is well known that a simply connected Riemann surface is biholomorphic to one of the Riemann sphere, the complex plane and the unit disc. And there are several results concerning the geometric characterization of these spaces. To show that some simply connected open Riemann surface is biholomorphic to the complex plane or the unit disc, it is sufficient to see that there exist non constant bounded subharmonic functions or not. But in the higher dimensional case, there is no uniformization theorem. By this reason to show that some complex manifold is biholomorphic to C^n or an open ball, we must construct a biholomorphic mapping directly.

The following problems are given by R. E. Greene and H. Wu in [1].

PROBLEM 1. If a complete Kähler manifold M is diffeomorphic to R^{2n} and its sectional curvature is non positive and larger than $-A/r^{2+\varepsilon}$ outside a compact set, then is M biholomorphic to C^n? Here r is the distance function from a fixed point of M and ε, A are some positive constants.

PROBLEM 2. If a complete Kähler manifold M is diffeomorphic to R^{2n} and its sectional curvature is non positive and smaller than $-A/r^2$ outside a compact set, then is M biholomorphic to a bounded domain?

In this paper we consider the above problems under very restrictive conditions, using a theorem of J. Milnor [4].

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