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ABELIAN VARIETIES ATTACHED TO CYCLES OF INTERMEDIATE DIMENSION

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0. Introduction

The group of cycles of codimension one algebraically equivalent to zero of a nonsingular projective variety modulo rational equivalence forms an abelian variety, i.e., the Picard variety. To the group of cycles of dimension zero and of degree zero, there corresponds an abelian variety, the Albanese variety. Similarly, Weil, Lieberman and Griffiths have attached complex tori to the cycles of intermediate dimension in the classical case. The aim of this article is to give a purely algebraic construction of such "intermediate Jacobian varieties."

We denote the group of cycles of codimension p of a nonsingular projective variety X modulo rational equivalence by $CH^p(X)$, the subgroup of $CH^p(X)$ consisting of cycles algebraically equivalent to zero by $A^p(X)$. Then $CH(X) = \oplus_{p>0} CH^p(X)$ has a ring structure, the Chow ring of X [19]. Lieberman has introduced an "axiom of intermediate Jacobian" [13]: For each nonsingular projective variety X and for each integer p $(1 \leq p \leq \dim X)$, there exist

(i) a subgroup $K^p(X)$ of $A^p(X)$ and
(ii) an abelian variety $J^p(X)$.

These should satisfy the following conditions:

(iii) $J^p(X)$ is the Albanese variety of X if $p = \dim X$.
(iv) There is an isomorphism $A^p(X)/K^p(X) \rightleftarrows J^p(X)$ of groups.
(v) Functoriality: for arbitrary varieties X and Y, and an element z of $CH^{p+q}(X \times Y)$, there exists an abelian variety homomorphism

$$[z]\colon J^{m-q}(X) \longrightarrow J^p(Y)$$

$(m = \dim X)$ such that the diagram

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