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HYPERBOLIC NONWANDERING SETS WITHOUT DENSE PERIODIC POINTS

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In this paper we give a negative answer to the problem which is suggested in [3]: if a nonwandering set Ω is hyperbolic, are the periodic points dense in Ω?

Newhouse and Palis proved that on two dimensional closed manifolds the answer is positive ([1], [2]).

Suppose that $f: M \to M$ is a diffeomorphism of a manifold M. A point $x \in M$ is a nonwandering point of f if for any neighbourhood $U \subset M$ of x there is a positive integer n such that $f^n(U) \cap U \neq \emptyset$. $\Omega = \{$nonwandering points of $f\}$ is called the nonwandering set of f. A point of $M - \Omega$ is a wandering point. A nonwandering set Ω of f is hyperbolic if Ω is compact and $TM|\Omega$ splits into a Whitney sum of Tf-invariant subbundles

$$TM|\Omega = E^s \oplus E^u ,$$

and there are $c > 0, 0 < \lambda < 1$ such that

$$\|Tf^n v\| \leqq c\lambda^n \|v\| \qquad \text{if } v \in E^s$$

and

$$\|Tf^{-n} v\| \leqq c\lambda^n \|v\| \qquad \text{if } v \in E^u$$

for $n > 0$.

We will prove the following.

THEOREM. *Suppose that M is a manifold with* $\dim M \geqq 4$. *Then there is a diffeomorphism $F: M \to M$ such that the nonwandering set Ω is hyperbolic but periodic points of F are not dense in Ω.*

Proof. **0.** An outline of Proof. To simplify the proof, we assume $\dim M = 4$. In 1 we construct an embedding of 2-dimensional disk $f: D$

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