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DISTANCE FUNCTIONS AND UMBILIC SUBMANIFOLDS OF HYPERBOLIC SPACE

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In 1972, Nomizu and Rodriguez [5] found the following characterization of the complete umbilic submanifolds of Euclidean space.

THEOREM A. *Let M^n , $n \geq 2$, be a connected, complete Riemannian manifold isometrically immersed in a Euclidean space E^m . Every Morse function of the form L_p has index 0 or n at all of its critical points if and only if M^n is embedded as a Euclidean n -subspace or a Euclidean n -sphere in E^m .*

Here L_p is the Euclidean distance function, $L_p(x) = |p - f(x)|^2$, where f is the immersion of M^n into E^m .

Cecil [2] characterized the metric spheres in hyperbolic space H^m in terms of hyperbolic distance functions L_p as follows.

THEOREM B. *Let M^n , $n \geq 2$, be a connected, compact, differentiable manifold immersed in H^m . Every Morse function L_p has exactly two critical points if and only if M^n is embedded as a metric n -sphere in H^m .*

In [2, p. 351], it was pointed out that the non-compact complete umbilic submanifolds in hyperbolic space could not be distinguished in terms of the distance functions alone. The purpose of this paper is to obtain the appropriate analogue of Theorem A for hyperbolic space through the introduction of two new natural classes of Morse functions.

Each of these classes is related to a type of umbilic hypersurface as follows. A function of the first new class will have as level sets a totally

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