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PROJECTIVE INVARIANT METRICS FOR EINSTEIN SPACES

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1. Introduction

In my recent paper [1], I associated a projectively invariant pseudo-distance d_M to every affinely connected manifold M and proved the following

THEOREM 1. *Let M be a Riemannian manifold with metric ds_M^2 and Ricci tensor* Ric_M *such that* $\mathrm{Ric}_M \leq -c^2 ds_M^2$. *Let δ_M be the Riemannian distance defined by ds_M^2. Then*

$$d_M(x, y) \geq \frac{2c}{\sqrt{n-1}}\delta_M(x, y) \qquad for\ \ x, y \in M .$$

The purpose of this paper is to show the following

THEOREM 2. *Let M be a complete Einstein manifold with*

$$\mathrm{Ric}_M = -c^2 ds_M^2 .$$

Then

$$d_M(x, y) = \frac{2c}{\sqrt{n-1}}\delta_M(x, y) \qquad for\ \ x, y \in M .$$

The following corollary has been known for some time [3], [4].

COROLLARY. *The projective transformations of a complete Einstein manifold with negative Ricci tensor are all isometries.*

Since Theorem 1 is not stated in [1] in the same manner as above, we shall first indicate how it can be derived from the results proved in [1].

We should remark that d_M vanishes identically if ds_M^2 is complete

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